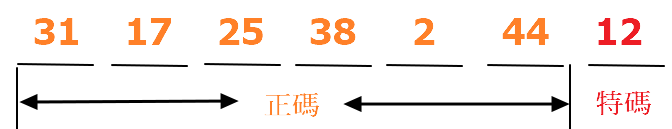
Combinatorics

Mark 6(六合彩) is constructed as follows:

In a pool of 1~49 numbers, 7 distinct numbers are chosen to place them in 7 positions. The 1st ~6th positions are called zheng-ma (ortho-code, 正碼) 1~6, and the 7th positions is called te-ma (special code, 特碼).



There is a category in Mark 6 call “connected codes”, it means players can choose few numbers as one bet. It contains few ways, like “3 out of 3”, or “2 out of 3”. In “3 out of 3”. Players choose 3 “distinct” numbers as one bet out of all 49 numbers, if all of them falling in the zheng-ma region after, then the player won with odds =663.

For example, a player chose 20 number, 25~44.

Namely, he bought C(20,3)=1140 bets, for each bet cost him 1 dollar. Since there are 4 numbers falling in the above zheng-ma region, hence he won C(4,3)=4 bets, total winning prize=663\*4=2652.

In “2 out of 3”, players choose multiple numbers out of all 49 numbers, every 3 distinct numbers form one bet, if two of them falling in the zheng-ma region, and another number NOT falling in the zheng-ma region, then the player won “2 out of 3, shot 2” with odds=21. In each bet, if all of them falling in the zheng-ma region, then the player won “3 out of 3, shot 3” with odds=109.

Question:

Now for a player, her choice goes as above. If each bet cost her 1 dollar, how many bets she won “2 out of 3, shot 2” and “3 out of 3, shot 3”? What’s the total prize she won?

Game Theory

In a game, there are 3 numbers, 1,2,3 for players to put their bets with Odds=3. The net profit table for player vs house goes as follows (assuming bet = 1 dollars):

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Player | | | |
| House |  | 1 | 2 | 3 |
| 1 | -2, 2 | 1,-1 | 1,-1 |
| 2 | 1,-1 | -2, 2 | 1,-1 |
| 3 | 1,-1 | 1,-1 | -2, 2 |

The number pair (X, Y) in grey area means: X house’ profit, Y player’s profit. Positive means gaining, negative means losing.

Question:

Now, there are 3 ordinary players: A, B, C. A bought number 1 with 10 dollars. B bought number 2 with 15 dollars. C bought number 3 with 20 dollars. And one rich guy D, he bought number 1 with 1000 dollars, number 2 with 1000 dollar, and number 3 with 950 dollars. Since house has a cheating mechanism, it can pre-calculate every outcome before the number being draw out, to decide which one is good for house. Which number(s) house chooses can prptect him from losing money?

Probability

The total capital of a house and a player is 10K and 1K, they play two sides game, big or small. The chance of winning for each side is 1/2, with odds=1.98. Each time, the player plays, it’s called one round. Every 500 rounds called a cycle. A player chooses his bet with Kelly’s formula to optimize his profits (for example, for the first round, the player can put 0~1000, and he chooses $250).

For 1000 cycles, estimate how many times the player is forced to leave (meaning broke, running out of money) before he can finish the cycle he is in (including the house broke)?

A few examples of player’s capital change during one cycle (500 rounds):

